Energy Policy Math: Interaction of greenhouse gas reduction mechanisms

Suppose emission reduction measure 1 reduces gross emissions E by fractional reduction r_1 so that reduced emissions $E'_1 = E(1 - r_1)$; and emission reduction measure 2 reduces gross emissions E by fraction r_2 so that reduced emissions $E'_2 = E(1 - r_2)$. What is the value of r_{1+2} such that the reduced emissions when applying both measures is

$$E'_{1+2} = E(1 - r_{1+2})\ ? \qquad (1)$$

We begin by calculating the value of E'_{1+2} from fundamental principles:

$$E'_{1+2} = E(1 - r_1)(1 - r_2)$$

then apply the good old FOIL method (remember high school?)

$$= E(1 - r_1 - r_2 + r_1 r_2)$$

and regroup terms

$$= E\big(1 - (r_1 + r_2 - r_1 r_2)\big) \qquad (2)$$

Now by comparison of equation (2) with equation (1) it is evident from simple inspection that

$$r_{1+2} = r_1 + r_2 - r_1 r_2\ . \qquad (3)$$

That is, the combined, fractional reduction of the two measures is the sum of the two independent reductions, minus the product of the two reductions.

Frequently in climate policy analysis we are asked to evaluate a suite of $\boldsymbol{n}$ policy measures that produce fractional greenhouse gas reductions $\boldsymbol{r_1}$, $\boldsymbol{r_2}$, ... $\boldsymbol{r_n}$ on different subsets $\boldsymbol{E_1}$, $\boldsymbol{E_2}$, ... $\boldsymbol{E_n}$ of the gross emissions $\boldsymbol{E}$. With matrix formulations we can solve equation (3) for all possible pairings of policies ($\boldsymbol{r_i}$, $\boldsymbol{r_j}$) by first creating a $\boldsymbol{n} \times \boldsymbol{n}$ matrix of all such combined reductions interactions

$$\mathbf{R} \equiv \begin{bmatrix} r_1 & \cdots & r_1 \\ \vdots & \ddots & \vdots \\ r_n & \cdots & r_n \end{bmatrix}$$

and then calculating the matrix of interaction terms $r_{i+j} = r_i + r_j - r_i r_j$ as

$$\mathbf{R}^{\boldsymbol{int}} = \mathbf{R} + \mathbf{R}^{\mathbf{T}} - \mathbf{R}(\mathbf{I} \circ \mathbf{R})$$

where $\mathbf{I} \circ \mathbf{R}$ represents the Hadamard product of the identity matrix $\mathbf{I}$ with $\mathbf{R}$. The terms on the diagonal of $\mathbf{R}^{\boldsymbol{int}}$ do not have physical meaning and may be ignored.

The matrix manipulation can be implemented easily in Excel using array formulae and operators. Notice that each resulting interaction term $\boldsymbol{r_{i+j}}$ must be applied only to the shared subset of emissions $E_i \cap E_j$, with the non-interacting terms $\boldsymbol{r_i}$ and $\boldsymbol{r_j}$ applying only to $E_i \setminus E_j$ and $E_j \setminus E_i$, respectively (where "\" is the relative complement operator).

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